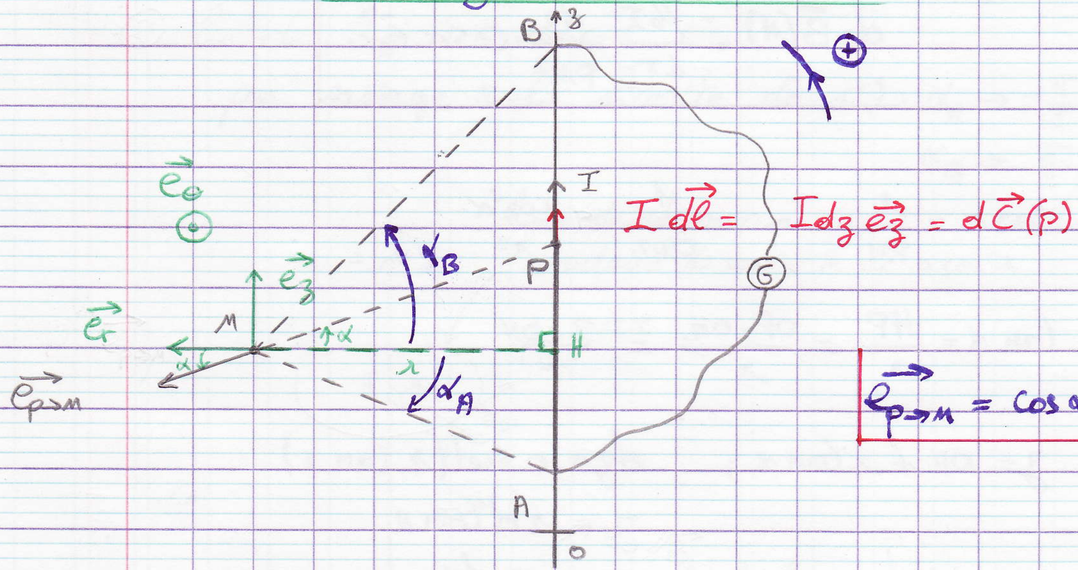
EM-4 (2) 04/06

IV - Cas du fil rectiligne infini

1°) Segment de courant



$$\boxed{\overrightarrow{e_{P\to M}} = \cos\alpha\,\vec{e_r} - \sin\alpha\,\vec{e_z}}$$

à proximité du fil

$$\vec{B}(M) = \oint_{\mathcal{C}} \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\overrightarrow{PM}}{PM^2} \simeq \int_{\text{fil } AB} d_P\vec{B}(M)$$

cf ②

$$d_P\vec{B}(M) = \frac{\mu_0}{4\pi}\, d\vec{C}(P) \times \frac{\overrightarrow{e_{P\to M}}}{PM^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\overrightarrow{e_{P\to M}}}{PM^2}$$

① la distribution du courant $\{AB\}$ est invariante $\forall^{te}$ rotation autour de Oz

$\hookrightarrow \forall M \in \mathcal{E} \quad \vec{B}(M) = \vec{B}(r,\theta,z) = \vec{B}(r,z) \quad (*)$

$(\pi) = (M, \vec{e_r}, \vec{e_z})$ est plan de symétrie des courants passant par M.

$\hookrightarrow \forall M \in \mathcal{E} \quad \vec{B}(M) \perp (\pi)$

$\hookrightarrow \vec{B}(M) = B(M)\,\vec{e_\theta} \quad (**)$

$\left.\begin{matrix}(*)\\(**)\end{matrix}\right\}\ \boxed{\vec{B}(M) = B(r,z)\,\vec{e_\theta}}$

② Loi de Biot & Savart :

$$d_P \vec{B}(M) = \frac{\mu_0}{4\pi} I dz \vec{e_z} \times \left(\frac{\cos\alpha \vec{e_r} - \sin\alpha \vec{e_z}}{PM^2}\right)$$

$$\boxed{d_P \vec{B}(M) = \frac{\mu_0 I}{4\pi} \frac{dz}{PM^2} \cos\alpha \, \vec{e_\theta}}$$

$$\cos\alpha = \frac{r}{PM} \longrightarrow \frac{1}{PM^2} = \frac{\cos^2\alpha}{r^2}$$

$$\tan\alpha = \frac{HP}{r} = \frac{OP - OH}{r} = \frac{z - OH}{r}$$

$z = OH + r\tan\alpha$; $\quad dz = 0 + d(r\tan\alpha)$

$$= r\, d\tan\alpha$$

$$= \frac{r\, d\alpha}{\cos^2\alpha}$$

$$\vec{B} = \int d_P \vec{B}(M) = \int_{P\in[AB]} \frac{\mu_0 I}{4\pi} \frac{r\, d\alpha}{\cancel{\cos^2\alpha}} \frac{\cancel{\cos^2\alpha}}{r^{\cancel{2}}} \cos\alpha \, \vec{e_\theta}$$

↳ vecteur défini en M indépt de P.

$$= \frac{\mu_0 I}{4\pi r}\left(\int_{\alpha_A}^{\alpha_B} \cos\alpha \, d\alpha\right) \vec{e_\theta}$$

$$= \frac{\mu_0 I}{4\pi r}\left[\sin\alpha\right]_{\alpha_A}^{\alpha_B} \vec{e_\theta}$$

$$\boxed{\vec{B}(M) = \frac{\mu_0 I}{4\pi r}\left(\sin\alpha_B - \sin\alpha_A\right) \vec{e_\theta}}$$

2°) Fil de courant infini

① $\mathcal{D}$ invariante

$\forall^{te}$ translat° selon Oz $\quad$ } $\forall M \in \mathcal{E}$

$\forall^{te}$ rotation autour de Oz $\quad$ } $\vec{B}(r,\theta,z) = \vec{B}(r)$

$(\pi) = (M, \vec{e_r}, \vec{e_z})$ est plan de sym. de $\mathcal{D}$ passant par M.

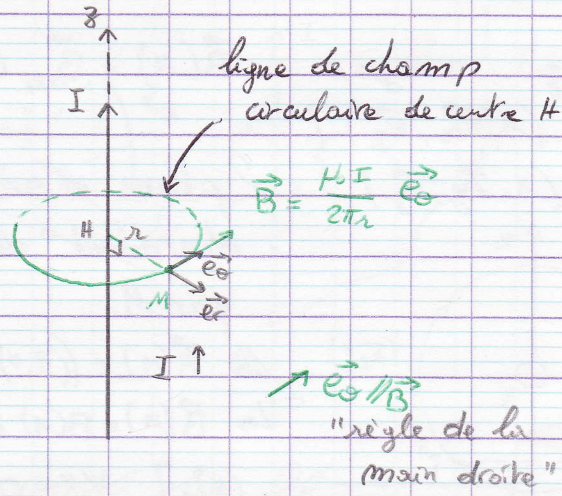
↳ $\forall M \in \mathcal{E}$: $\vec{B}(M) = B(M)\,\vec{e_\theta}$

CCL : $\boxed{\forall M \in \mathcal{E} : \vec{B}(M) = B(r)\vec{e_\theta}}$

① Méthode la + rapide : Thm d'Ampère cf EMS

②' loi de Biot & Savart :

par passage à la limite de 1°) : $\alpha_A \to -\frac{\pi}{2}$, $\alpha_B \to \frac{\pi}{2}$ $\quad \boxed{B(M) = \frac{\mu_0 I}{4\pi}\vec{e_\theta}}$



ordre de grandeur :

$I = 10\,A$ (!!)

$r = R = 10\,cm$

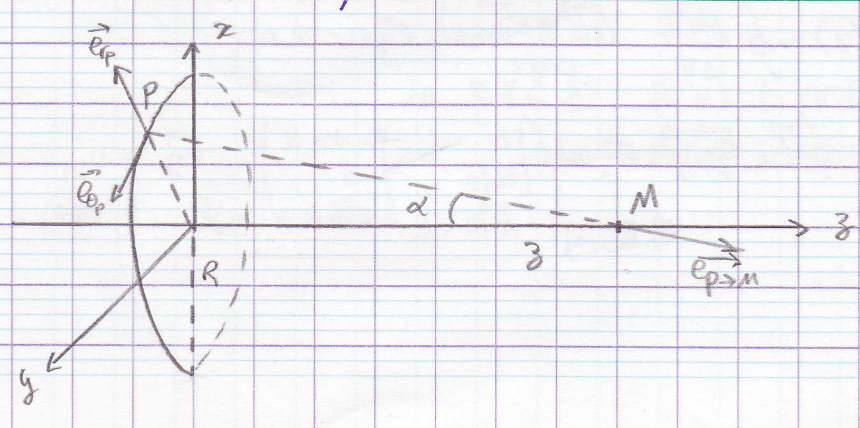
↳ $B = \underbrace{\frac{\mu_0}{4\pi}}\frac{2I}{r} = \underbrace{10^{-7}} \cdot \frac{2.10}{10.10^{-2}}$

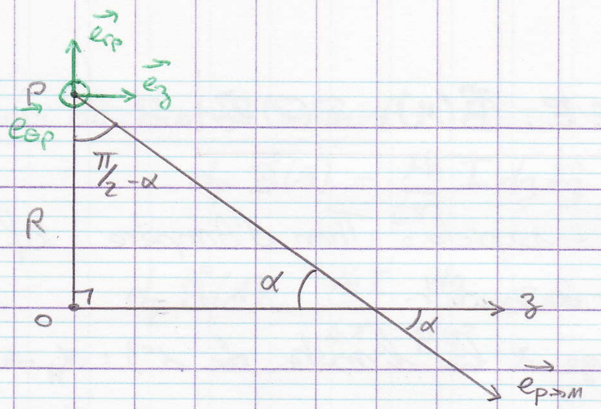
$\boxed{B = 2.10^{-5}\,T}$ $\quad$ uSI($\vec{B}$) = T (le tesla)

chp faible ≈ B terrestre.

IV - Calculs de champ $\vec{B}$

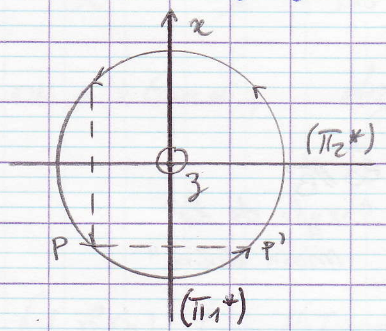
1°) Disque circulaire de courant





① • $\forall M \in Oz \quad x = y = 0 \quad \vec{B}(M) = \vec{B}_{axe}(z) = \vec{B}(z) \quad (*)$

• $\left.\begin{array}{l}(\Pi_1^*) = (M, \vec{e_x}, \vec{e_z}) \\ (\Pi_2^*) = (M, \vec{e_y}, \vec{e_z})\end{array}\right\}$ 2 plans d'antisymétrie de la distribut° de courants passant par M.



↳ $\vec{B}(M) \subset ((\Pi_1^*) \cap (\Pi_2^*)) = (M, z) = (Oz)$

↳ $\vec{B}(M) = B(M)\,\vec{e_z} \quad (**)$

$\left.\begin{array}{l}(*) \\ (**)\end{array}\right\}$ $\underline{\forall M \in Oz}$ $\boxed{\vec{B}(M) = B(z)\,\vec{e_z}}$

② Loi de Biot & Savart

$$\vec{B}(M) = \int_{P\in D} d_P\vec{B}(M) = \int_{P\in D} \frac{\mu_0}{4\pi}\,\frac{d\vec{C}(P) \times \vec{e_{P\to M}}}{PM^2}$$

$$= \oint \frac{\mu_0}{4\pi}\,\frac{I\,\vec{dl} \times \vec{e_{P\to M}}}{PM^2}$$

$$= \underbrace{B(z)}_{\text{ce qu'on cherche}}\,\vec{e_z}$$

$$B(z) = \vec{B}\cdot\vec{e_z} = \left(\oint \frac{\mu_0}{4\pi}\,\frac{I\,\vec{dl} \times \vec{e_{P\to M}}}{PM^2}\right)\cdot\vec{e_z}$$

$$B(z) = \oint \frac{\mu_0 I}{4\pi}\,\frac{(\vec{dl} \times \vec{e_{P\to M}})\cdot\vec{e_z}}{PM^2}$$

$$(\vec{dl} \times \vec{e_{P\to M}})\cdot\vec{e_z} = \left(\begin{vmatrix}0\\ R\,d\theta\\ 0\end{vmatrix} \times \begin{vmatrix}-\sin\alpha\\ 0\\ \cos\alpha\end{vmatrix}\right)_{\vec{e_\rho}, \vec{e_{\theta P}}, \vec{e_z}}\cdot\vec{e_z}$$