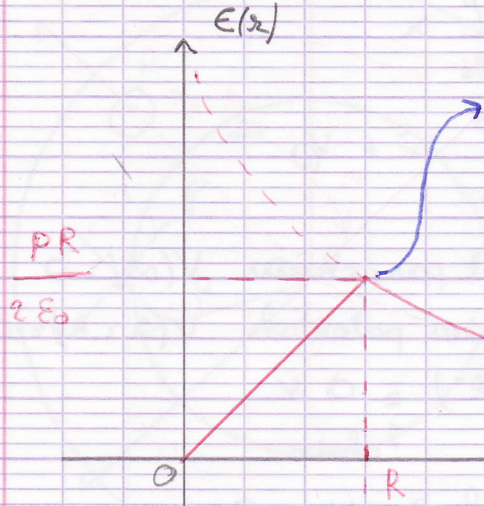
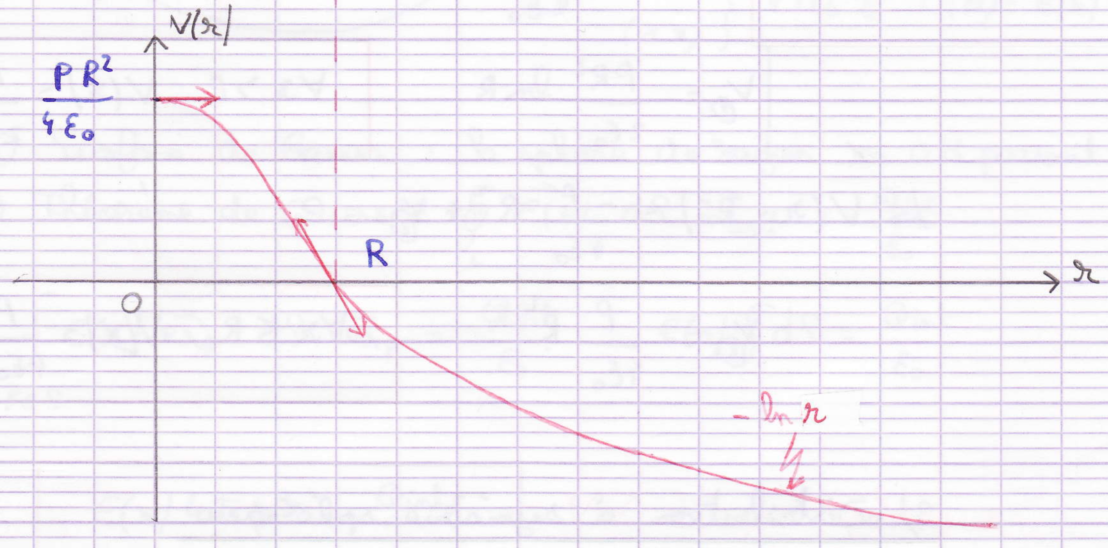
$$\boxed{\begin{array}{ll} \forall M \,/\, r > R & \vec{E}(M) = \dfrac{\rho}{2\varepsilon_0}\dfrac{R^2}{r}\vec{e_r} \\[2ex] \forall M \,/\, r < R & \vec{E}(M) = \dfrac{\rho}{2\varepsilon_0}\, r\, \vec{e_r} \end{array}}$$

<u>Prop</u>: $E(r)$ est continu à la traversée de $\mathcal{D}$ volumique $E(R^+) = E(R^-)$





④ V ? $\vec{E}(M) = -\vec{\mathrm{grad}}\, V$ $\quad E(r)\vec{e_r} = - \left|\begin{array}{l} \dfrac{\partial V}{\partial r} \\[1ex] \cancel{\dfrac{1}{r}\dfrac{\partial V}{\partial \theta}} \\[1ex] \cancel{\dfrac{\partial V}{\partial z}} \end{array}\right._{(\vec{e_r},\vec{e_\theta},\vec{e_z})} = -\dfrac{dV}{dr}\vec{e_r}$

$$\frac{dV}{dr} = -E(r)$$

1^{er} cas : $r > R$ $\quad \frac{dV}{dr} = \frac{-\rho}{2\varepsilon_0} \frac{R^2}{r}$

$$V = \frac{-\rho}{2\varepsilon_0} R^2 \ln r + V_1$$

2^e cas : $r < R$ $\quad \frac{dV}{dr} = -\frac{\rho}{2\varepsilon_0} r$

$$V = \frac{-\rho}{4\varepsilon_0} r^2 + V_2$$

D illimitée dc on ne peut pas choisir $\cancel{V(\infty)} = 0$

↳ choix de l'origine des potentiels $V(r = R) = 0V$

↳ $V(r = R^+) = V(r = R^-) \equiv 0\,V$ $\underset{\uparrow \text{choix}}{}$

↳ $V(r = R^+) = -\frac{\rho R^2}{2\varepsilon_0} \ln R + V_1 = 0$

$$V_1 = \frac{\rho R^2}{2\varepsilon_0} \ln R$$

$$\boxed{\forall r > R \quad V(r) = \frac{\rho R^2}{2\varepsilon_0} \ln \frac{R}{r}}$$

↳ $V(r = R^-) = \frac{-\rho}{4\varepsilon_0} R^2 + V_2 = 0$

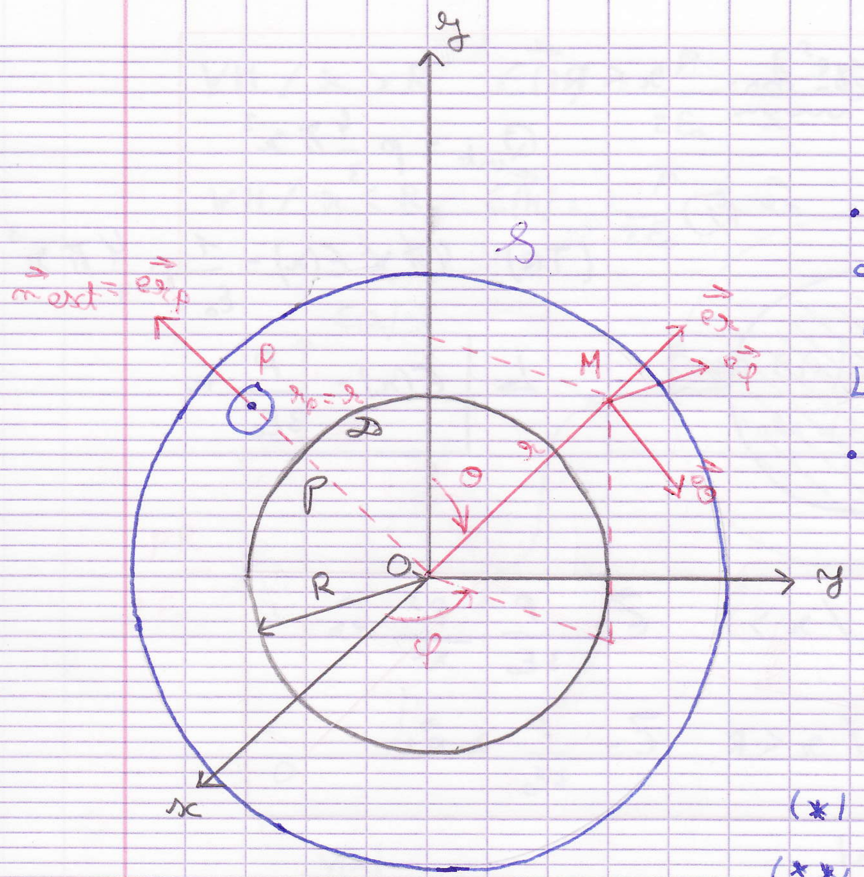
$$V_2 = \frac{\rho}{4\varepsilon_0} R^2$$

$$\boxed{\forall r < R \quad V(r) = \frac{\rho}{4\varepsilon_0} (R^2 - r^2)}$$

3) Distribution à symétrie sphérique

a.

Hyp : D sphère | unif^t chargée en volume
de rayon R



① Base adaptée : sphérique

• $\mathcal{D}$ est invariante $\forall^{te}$ rotat° autour de n'importe quel axe passant par O :

↳ $\vec{E}(M) = E(r, \theta, \varphi) = E(r)$ (*)

• $\Pi_1 = (M, \vec{e_r}, \vec{e_\theta})$; $\Pi_2 = (M, \vec{e_r}, \vec{e_\varphi})$ } 2 plans de symétries des charges passant par M

↳ $\vec{E}(M) \subset [(\Pi_1) \cap (\Pi_2)] = (M, \vec{e_r})$

$\vec{E}(M) = E(M)\vec{e_r}$ (**)

(*) (**) } $\boxed{\forall M \in \mathcal{E} \quad \vec{E}(M) = E(r)\vec{e_r}}$

② Surface de Gauss : la sphère de rayon $r = OM$ passant par M

③ Théorème de Gauss : $\oint_S \vec{E}(P).dS.\vec{n}_{ext} = \frac{Q_{int}}{\varepsilon_0}$

$$\int_{P \in S} E(r)\,\vec{e}_{r_P}.dS.\vec{e}_{r_P} = \frac{Q_{int}}{\varepsilon_0} \qquad E(r)\int dS = \frac{Q_{int}}{\varepsilon_0}$$

$$\boxed{E(r)\,4\pi r^2 = \frac{Q_{int}}{\varepsilon_0}}$$ $\forall \mathcal{D}$ à sym sphérique

Rappel
$V_{sph} = \frac{4}{3}\pi R^3$
$S_{sph} = 4\pi R^2$

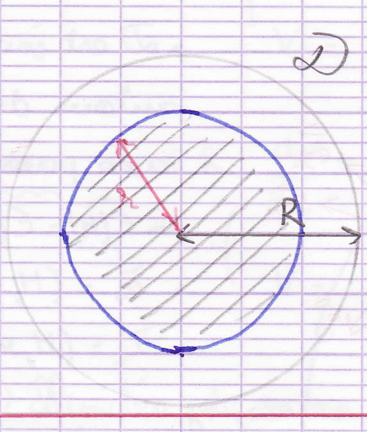
• Ici   1er cas : $r > R$ $Q_{int} = Q_{totale} = \rho V_{\mathcal{D}} = \rho \frac{4}{3}\pi R^3$

(TG) → $E(r)\,4\pi r^2 = \frac{1}{\varepsilon_0}\rho\frac{4}{3}\pi R^3$

↳ $E(r) = \frac{\rho}{3\varepsilon_0}\frac{R^3}{r^2}$

2e cas : $r < R$

$$Q_{int} = \rho \frac{4}{3}\pi r^3$$



(TG) $\cancel{4\pi} r^2 E(r) = \frac{1}{\varepsilon_0}\rho\frac{4}{3}\cancel{\pi} r^3$

↳ $\boxed{E(r) = \frac{\rho}{3\varepsilon_0} r}$

$$\boxed{\begin{array}{ll} \forall M \in \mathcal{E} \,/\, r > R & \vec{E} = \frac{\rho}{3\varepsilon_0}\frac{R^3}{r^2}\vec{e_r} \\ \forall M \in \mathcal{E} \,/\, r < R & \vec{E} = \frac{\rho}{3\varepsilon_0} r\,\vec{e_r} \end{array}}$$