Ex EM3-5 :

$\mathcal{D}$ à la sym. sphérique $\mathcal{D}(O, R, \rho(r))$

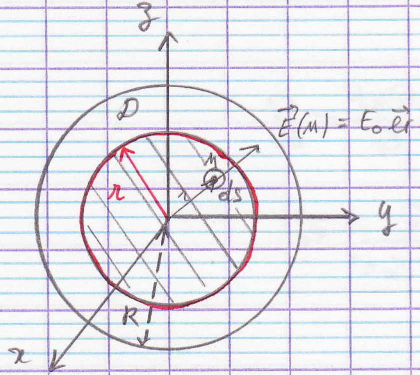
↳ en base sphérique

$$\boxed{\forall M \in \mathcal{E} : \vec{E}(M) = E(r)\,\vec{e_r}}$$

on impose $\vec{E}_{int} = E_0\,\vec{e_r}$

$$\boxed{\forall r \le R : \quad E(r) = E(0)}$$

Q: $\rho(r)$?



Thm de Gauss pour sphère S de rayon $r < R$

$$\oint_S \vec{E}(P)\cdot dS\,\vec{n}_{ext} = \frac{Q_{int}}{\varepsilon_0}$$

$$\oint_S E_0\,\underbrace{\vec{e_r}\cdot dS\,\vec{e_r}}_{1} = \frac{Q_{int}}{\varepsilon_0}$$

$$E_0 \oint_S dS = \frac{Q_{int}}{\varepsilon_0}$$

(TG) $\boxed{E_0 \cdot 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0}}$ $= \frac{1}{\varepsilon_0}\int dq(P)$

$$= \frac{1}{\varepsilon_0}\iiint \rho(P)\,d\tau = \frac{1}{\varepsilon_0}\int_0^r \rho(r) r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$Q_{int} = \int dq = \int_0^r \rho(r')\,r'^2\,dr' \underbrace{\int_0^\pi \sin\theta\, d\theta}_{2} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi}$$

$$\boxed{Q_{int} = 4\pi \int_0^r \rho(r')\,r'^2\,dr'}$$

(TG) → $E_0 r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho(r')\,r'^2\,dr'$

d/dr ↓

$$E_0\, 2\cancel{r} = \frac{1}{\varepsilon_0}\rho(r)\,r^{\cancel{2}}$$

$$\boxed{\rho(r) = \frac{2\varepsilon_0 E_0}{r}}$$

Q2 : charge totale de $\mathcal{D}$?

$$Q \equiv \int dq = \int \rho(r)\, r^2 dr \sin\theta\, d\theta\, d\varphi$$

$$= 4\pi \int_0^R \rho(r)\, r^2 dr$$

$$= 4\pi \int_0^R 2\varepsilon_0 E_0\, r\, dr$$

$$= 4\pi \,.\, \cancel{2}\varepsilon_0 E_0 \frac{R^2}{\cancel{2}}$$

$$\boxed{Q = 4\pi R^2 \varepsilon_0 E_0}$$

(TG) pour $r > R$

$$E(r)\, 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$\boxed{\vec{E}_{ext} = \frac{Q}{4\pi\varepsilon_0 r^2}\, \vec{e_r}}$$