$$E(y) = \int_{-\pi/2} \frac{\lambda dl}{4\pi\varepsilon_0} \times \frac{-R\cos\theta}{PM^3} + \int_{\pi/2} \frac{-\lambda dl}{4\pi\varepsilon_0} \frac{-R\cos\theta}{PM^3}$$

$$= \frac{-\lambda R^2}{4\pi\varepsilon_0} \frac{1}{PM^3} \int_{-\frac{\pi}{2}}^{\pi/2} \cos\theta \, d\theta + \frac{\lambda R^2}{4\pi\varepsilon_0} \frac{1}{PM^3} \int_{\pi/2}^{3\pi/2} \cos\theta \, d\theta$$

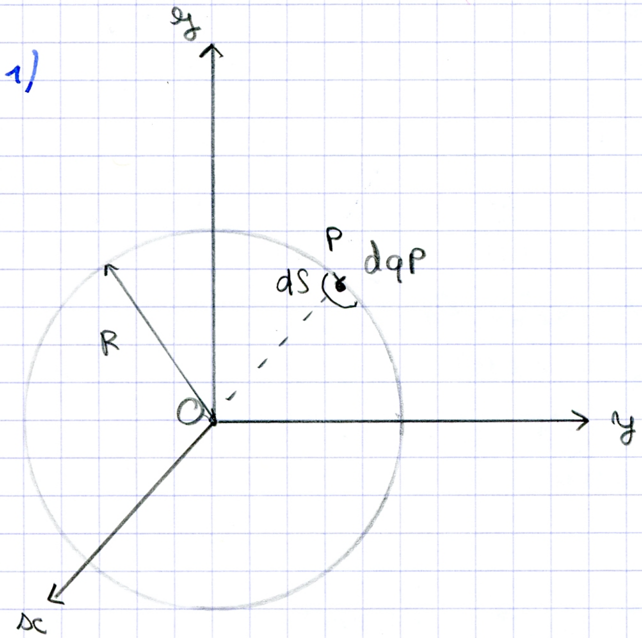
$$= \frac{-\lambda}{4\pi\varepsilon_0} \frac{R^2}{PM^3} \underbrace{\left[\underbrace{[\sin\theta]_{-\pi/2}^{\pi/2}}_{2} - \underbrace{[\sin\theta]_{\pi/2}^{3\pi/2}}_{(-2)} \right]}_{4}$$

$$E(y) = \frac{-\lambda}{\pi\varepsilon_0} \frac{R^2}{PM^3}$$

↳ soit
$$\boxed{\vec{E}(M) = \frac{-\lambda}{\pi\varepsilon_0} \frac{R^2}{PM^3} \vec{e}_x = \frac{-\lambda}{\pi\varepsilon_0} \frac{R^2}{(R^2+y^2)^{3/2}} \vec{e}_x}$$

Ex EM2_5 :

1)



Potentiel électrostatique en O

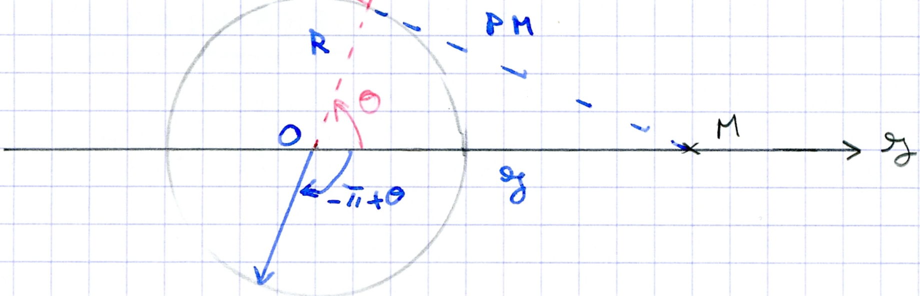
$$V(O) = \int \frac{dq(P)}{4\pi\varepsilon_0} \frac{1}{PO} = \int \frac{\sigma dS}{4\pi\varepsilon_0} \frac{1}{R}$$

Loi de Coulomb

$$= \frac{\sigma}{4\pi\varepsilon_0 R} \int dS$$

$$V(O) = \frac{\sigma S}{4\pi\varepsilon_0} \frac{1}{R} = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 R}$$

$$\boxed{V(O) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R} = \frac{\sigma}{\varepsilon_0} R}$$



$$V(M) = \int d_P V(M) = \int \frac{dq(P)}{4\pi\varepsilon_0}\,\frac{1}{PM} = \int \frac{\sigma\, dS}{4\pi\varepsilon_0}\,\frac{1}{PM} = \int \frac{\sigma R^2 \sin\theta\, d\theta\, d\varphi}{4\pi\varepsilon_0\, PM}$$

car $d\overrightarrow{OP} = \left|\begin{array}{l} dr \\ r\,d\theta \\ r\sin\theta\, d\varphi \end{array}\right.$ $(\vec{e_r}, \vec{e_\theta}, \vec{e_\varphi})$

$$\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM}$$

$$PM = \sqrt{\|\overrightarrow{PM}\|^2} = \sqrt{\overrightarrow{PM}\cdot\overrightarrow{PM}}$$
$$= \sqrt{PO^2 + OM^2 + 2\,\overrightarrow{PO}\cdot\overrightarrow{OM}}$$
$$= \sqrt{R^2 + y^2 + 2Ry\,\underbrace{\cos(\overrightarrow{PO},\overrightarrow{OM})}_{-\cos\theta}}$$

$\hookrightarrow PM = \sqrt{R^2 + y^2 - 2yR\cos\theta}$

$$V(M) = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} \frac{\sigma R^2}{4\pi\varepsilon_0}\,\frac{\sin\theta\, d\theta}{\sqrt{R^2+y^2-2yR\cos\theta}}\, d\varphi$$

$$= \frac{\sigma R}{4\pi\varepsilon_0}\int_0^{\pi} \frac{R\, d(-\cos\theta)}{\sqrt{R^2+y^2-2yR\cos\theta}}\int_0^{2\pi} d\varphi$$

$$V(M) = \frac{\sigma R}{4\pi\varepsilon_0}\,\frac{2\pi}{y}\left(\sqrt{R^2+y^2+2yR} - \sqrt{R^2+y^2-2yR}\right)$$

$$= \frac{\sigma R}{2\varepsilon_0}\,\frac{1}{y}\left(\sqrt{(R+y)^2} - \sqrt{(R-y)^2}\right)$$

$$\boxed{V(M) = \frac{\sigma R}{2\varepsilon_0}\,\frac{1}{y}\left(|R+y| - |R-y|\right)}$$