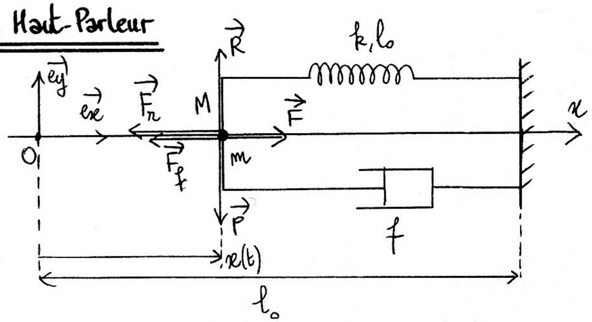
EXMS.4 Modélisation d'un Haut-Parleur



1) $\{M, m\}$

PFD : $m\vec{a}_{M/R_g} = \underbrace{\vec{P}+\vec{R}}_{\vec{0} \text{ car mvmt plan}} + \vec{F} + \vec{F_r} + \vec{F_f}$

$\vec{F} = K\,i(t)\,\vec{e_x}$

$\vec{F_r} = k(l-l_0)\vec{e_x} = -kx\,\vec{e_x}$

$\vec{F_f} = -f\dot{x}\,\vec{e_x}$

↳ en projection selon $\vec{e_x}$: $m\ddot{x} = Ki(t) - kx - f\dot{x}$

d'où $\boxed{\ddot{x} + \frac{f}{m}\dot{x} + \frac{k}{m}x = \frac{K}{m} I_m \cos\omega t}$

2) $\boxed{\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x = \frac{K}{m} I_m \cos\omega t}$ $\boxed{\omega_0 = \sqrt{\frac{k}{m}}}$ $\boxed{Q = \frac{m\omega_0}{f} = \frac{\sqrt{km}}{f}}$

AN on veut $Q = \frac{1}{\sqrt{2}}$ d'où $\boxed{f = \frac{\sqrt{km}}{Q} = \sqrt{2km} \cong 17{,}3 \text{ kg.s}^{-1}}$

3) $\begin{cases} x(t) = X_m\cos(\omega t + \varphi) \\ \underline{X} = X_m e^{j\varphi} \end{cases} \xrightarrow{\text{passage en complexe}} \underline{X}\left(\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}\right) = \frac{KI_m}{m}$

d'où $\underline{X} = \dfrac{\frac{KI_m}{m}}{\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}} = \dfrac{KI_m}{m\omega_0^2}\,\dfrac{1}{1 - \frac{\omega^2}{\omega_0^2} + j\frac{\omega}{Q\omega_0}} = \dfrac{-jKI_m}{m\omega_0^2}\,\dfrac{1}{\frac{\omega}{Q\omega_0} + j\left(\frac{\omega^2}{\omega_0^2} - 1\right)}$

↳ $\boxed{X_m = \dfrac{KI_m}{m\omega_0^2}\,\dfrac{1}{\sqrt{\left(\frac{\omega^2}{\omega_0^2} - 1\right)^2 + \frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}}}}$ et $\varphi = \arg\underline{X} = -\frac{\pi}{2} - \arctan\dfrac{\frac{\omega^2}{\omega_0^2} - 1}{\frac{\omega}{Q\omega_0}}$

soit $\boxed{\varphi = -\frac{\pi}{2} - \arctan Q\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]}$

AN : $\omega_0 = \sqrt{\frac{k}{m}} \cong 1\,225 \text{ rad.s}^{-1}$ et $\omega = 6\,280 \text{ rad.s}^{-1}$

$\Rightarrow$ $\boxed{X_m \cong 0{,}5 \text{ mm} \quad \text{et} \quad \varphi = -164° = -2{,}86 \text{ rad.}}$

↳ $x(t) = 0{,}5.10^{-3}\cos(6\,280t - 2{,}86)$ (x en mètres)

4) Pour $Q = \frac{1}{\sqrt{2}}$

on a : $X_m = \dfrac{KI_m}{m\omega_0^2}\,\dfrac{1}{\sqrt{\left(\frac{\omega^2}{\omega_0^2}\right)^2 - \cancel{2\frac{\omega^2}{\omega_0^2}} + 1 + \cancel{2\frac{\omega^2}{\omega_0^2}}}} \Rightarrow X_m = \dfrac{\frac{KI_m}{m\omega_0^2}}{\sqrt{1 + \frac{\omega^4}{\omega_0^4}}}$

Il s'agit d'un filtre Passe bas : sa bande passante est $[0, \omega_c]$

avec ω_c, la pulsation de coupure pour laquelle :

$X_m = \dfrac{X_m(max)}{\sqrt{2}}$

soit $\boxed{\omega_c = \omega_0}$