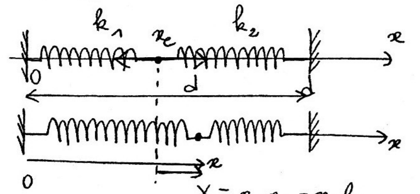
EXM4.2



$X \equiv x - x_e = x - \ell_1$

1) à l'équilibre :

$$\begin{cases} 0 = -k_1(\ell_1 - \ell_0) + k_2(\ell_2 - \ell_0) & ① \\ \ell_1 + \ell_2 = d \to \ell_2 = d - \ell_1 \end{cases}$$

$$\to \quad 0 = (-k_1 - k_2)\ell_1 + (k_1 - k_2)\ell_0 + k_2 d$$

$$\boxed{\ell_1 = \frac{(k_1 - k_2)\ell_0 + k_2 d}{k_1 + k_2}}$$

AN : $\boxed{\ell_1 = 35\text{ cm} \qquad \ell_2 = 25\text{ cm}}$ $(d - \ell_1 = 60 - 35\text{ cm})$

2) PFD :

$$\begin{vmatrix} m\ddot{x} = -k_1(x - \ell_0) + k_2(d - x - \ell_0) & ② \\ 0 = -k_1(\ell_1 - \ell_0) + k_2(\ell_2 - \ell_0) & ① \end{vmatrix} \qquad \ell_1 = x_{eq} \quad \ell_2 = d - x_{eq}$$

② - ① $\quad m\ddot{x} = -k_1(x - x_{eq}) + k_2(-x + x_{eq})$

$X \equiv x - x_{eq} \to m\ddot{x} = -k_1 X + k_2(-X) \to \boxed{\ddot{X} + \frac{k_1 + k_2}{m} X = 0}$

$\dot{X} = \dot{x}$

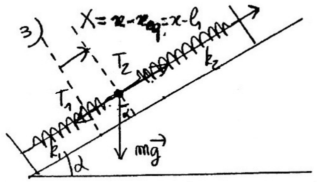
↳ ainsi, l'origine des abscisses est la posit° d'équilibre

$\omega_0^2 = \frac{k_1 + k_2}{m} \Rightarrow \quad \ddot{X} + \omega_0^2 X = 0$

$\to \boxed{T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k_1 + k_2}} = 0{,}63\text{ s}} \qquad \boxed{\mathcal{E}_m = \frac{1}{2}(k_1 + k_2)a_0^2}$

$$\mathcal{E}_m = \mathcal{E}_p + \mathcal{E}_k = \frac{1}{2}(k_1 + k_2)X^2 + \frac{1}{2}m\dot{X}^2 \underset{\text{f's conservatives}}{=} \text{cte} \underset{t=0}{=} \frac{1}{2}(k_1 + k_2)a_0^2$$

3)



ⓐ A l'équilibre :

$$\begin{cases} 0 = -k_1(\ell_1 - \ell_0) - mg\sin\alpha + k_2(\ell_2 - \ell_0) & ① \\ \ell_1 + \ell_2 = d \to \ell_2 = d - \ell_1 \end{cases}$$

↳ $0 = (-k_1 - k_2)\ell_1 + (k_1 - k_2)\ell_0 + k_2 d - mg\sin\alpha$

$$\boxed{\ell_1 = \frac{(k_1 - k_2)\ell_0 + k_2 d - mg\sin\alpha}{k_1 + k_2}} \qquad \text{AN} \quad \begin{Vmatrix} \ell_1 = 34{,}95\text{ cm} \\ \ell_2 = 25{,}05\text{ cm} \end{Vmatrix}$$

ⓑ PFD $\quad m\ddot{x} = -k_1(x - \ell_0) - mg\sin\alpha + k_2(d - x - \ell_0) \quad ②$

② - ① $\quad m\ddot{x} = -k_1(x - \ell_1) + k_2(d - x - \ell_2) = -k_1(x - \ell_1) + k_2(-x + \ell_1)$

$X \equiv x - \ell_1 \to \boxed{\ddot{X} + \frac{k_1 + k_2}{m} X = 0 \qquad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}}$ comme précédemment !

$\mathcal{E}_m = \text{cte} = \mathcal{E}_k + \mathcal{E}_{p\acute{e}l} + \mathcal{E}_{pg} = \frac{1}{2}m\dot{X}^2 + \frac{1}{2}(k_1 + k_2)X^2 + mg\,x\sin\alpha$