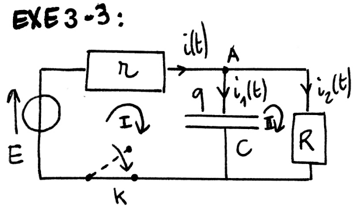
EXE3-3 :



Loi des Mailles (I) : $+E - ri - \frac{q}{C} = 0$ (1)

Loi des Mailles (II) : $-Ri_2 + \frac{q}{C} = 0$ (2)

Loi des noeuds en A : $i = i_1 + i_2$ (3)

$i_1 = \frac{dq}{dt}$ (4)

(1) → $i = \frac{-q}{rC} + \frac{E}{r}$ (1)   (2) → $i_2 = \frac{q}{RC}$ (3) → $-\frac{q}{rC} + \frac{E}{r} = \frac{dq}{dt} + \frac{q}{RC}$

Soit : $\frac{dq}{dt} + \frac{1}{C}\left(\frac{1}{R}+\frac{1}{r}\right)q = \frac{E}{r}$ $\boxed{\tau = \frac{CRr}{R+r} \quad \frac{dq}{dt} + \frac{q}{\tau} = \frac{E}{r}}$

$q(t) = \frac{E\tau}{r} + A\exp\frac{-t}{\tau} = \frac{ECR}{R+r} + A\exp\frac{-t}{\tau}$

CI : t=0 $q(0^-) = 0 = q(0^+) = \frac{CRE}{r+R} + A$ → $\boxed{q(t) = \frac{ECR}{R+r}\left(1 - \exp\frac{-t}{\tau}\right)}$

(4) → $i_1(t) = \frac{dq}{dt} = \frac{ECR}{R+r}\frac{1}{\tau}\exp\frac{-t}{\tau}$ → $\boxed{i_1(t) = \frac{E}{r}\exp\frac{-t}{\tau}}$

(2) → $i_2(t) = \frac{q}{RC}$ → $\boxed{i_2(t) = \frac{E}{R+r}\left(1 - \exp\frac{-t}{\tau}\right)}$

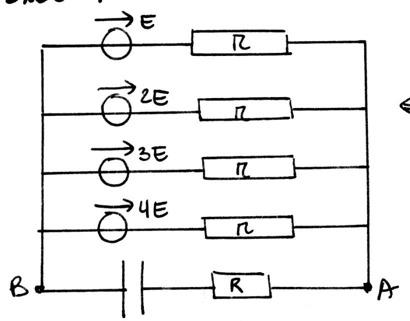
(3) → $i = i_1 + i_2$ → $\boxed{i(t) = \frac{E}{R+r}\left(1 + \frac{R}{r}\exp\frac{-t}{\tau}\right)}$

2) Énergie stockée dans le condensateur à la date t_1 : $W_C = \frac{1}{2}\frac{q_1^2}{C}$

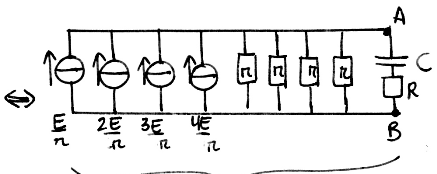
$W_C = \frac{C}{2}\left[\frac{RE}{R+r}\left(1 - \exp\frac{-t_1}{\tau}\right)\right]^2$

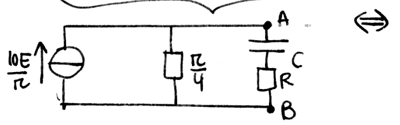
3) $W_G = W_C + W_J$ avec $W_G = \int_0^{t_1} E.i\,dt$ $W_J = \int_0^{t_1}\left(ri^2 + Ri_2^2\right)dt$

EXE3-4

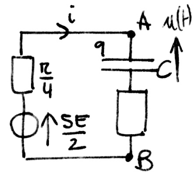


⇔





⇔



Loi des Mailles → $\frac{5E}{2} - \frac{r}{4}i - Ri - \frac{q}{C} = 0$; $u = \frac{q}{C}$ $i = \frac{dq}{dt}$ → $\left(R + \frac{r}{4}\right)\frac{di}{dt} + \frac{i}{C} = 0$

$\boxed{\frac{1}{\tau} = \frac{1}{C\left(R+\frac{r}{4}\right)} \Rightarrow \frac{di}{dt} + \frac{i}{\tau} = 0}$

$i(t) = A\exp\frac{-t}{\tau}$

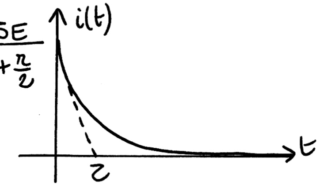
$u(0^-) = 0 = u(0^+) = \frac{q(0^+)}{C} = 0$ → Loi des Mailles à t = 0 : $\frac{5E}{2} - \left(R + \frac{r}{4}\right)i(0^+) = 0$

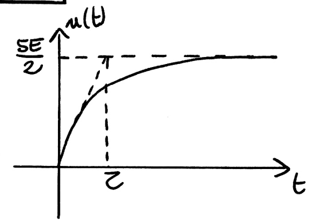
↳ $i(0^+) = \frac{5E}{2R+\frac{r}{2}} = A$ → $\boxed{i(t) = \frac{5E}{2R+\frac{r}{2}}\exp\frac{-t}{\tau} \quad \text{avec } \tau = C\left(R+\frac{r}{4}\right)}$

$u(t) = \frac{q}{C} = \frac{1}{C}\int i(t')dt' = \frac{5E}{C\left(2R+\frac{r}{2}\right)}(-\tau)\exp\frac{-t}{\tau} + K = \frac{-5E\,\cancel{C}\cancel{\left(R+\frac{r}{4}\right)}}{\cancel{C}\,2\cancel{\left(R+\frac{r}{4}\right)}}\exp\frac{-t}{\tau} + K$

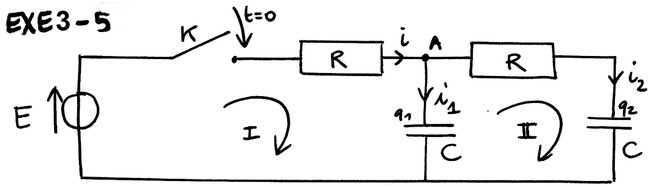
$u(0^+) = 0 = -\frac{5E}{2} + K$ → $\boxed{u(t) = \frac{5E}{2}\left(1 - \exp\frac{-t}{\tau}\right)}$

$\frac{10E}{4R+r} = \frac{5E}{2R+\frac{r}{2}}$





EXE3-5



1) Loi des noeuds en A : $i = i_1 + i_2$ (1)

$i_1 = \frac{dq_1}{dt}$ (2) $i_2 = \frac{dq_2}{dt}$ (3)

Loi des Mailles I : $E - Ri - \frac{q_1}{C} = 0$ (4)

⇓

$i = \frac{E}{R} - \frac{q_1}{RC}$ (4)

Loi des Mailles II : $\frac{q_1}{C} - Ri_2 - \frac{q_2}{C} = 0$ (5)

⇓

$q_1 = CRi_2 + q_2$ (5)

⇓

(2) $i_1 = \frac{dq_1}{dt} = CR\frac{d(i - i_1)}{dt} + \frac{dq_2}{dt}$

↑ (1)

(2) (4) → $i_1 = CR\frac{d}{dt}\left(\frac{E}{R} - \frac{q_1}{RC}\right) - CR\frac{di_1}{dt} + i_2$

$i_1 = 0 - \frac{dq_1}{dt} - CR\frac{di_1}{dt} + (i - i_1) = 0 - i_1 - CR\frac{di_1}{dt} + \frac{E}{R} - \frac{q_1}{RC} - i_1$

(2) → $\boxed{3 i_1 + CR \frac{di_1}{dt} + \frac{q_1}{RC} = \frac{E}{R}} \xrightarrow{\frac{d}{dt}} CR \frac{d^2 i_1}{dt^2} + 3\frac{di_1}{dt} + \frac{1}{RC} i_1 = 0$ (*)

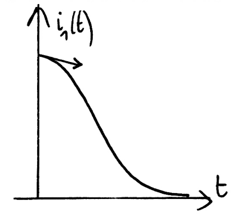
↳ $$\boxed{\begin{array}{l} \frac{d^2 i_1}{dt^2} + \frac{3}{RC}\frac{di_1}{dt} + \frac{i_1}{R^2C^2} = 0 \\ \frac{d^2 i_1}{dt^2} + \frac{\omega_0}{Q}\frac{di_1}{dt} + \omega_0^2 i_1 = 0 \end{array}}$$

$\omega_0 = \frac{1}{RC}$ $\quad \frac{\omega_0}{Q} = \frac{3}{RC} \Rightarrow Q = \frac{1}{3} < \frac{1}{2}$

↳ régime apériodique

2) Conditions Initiales : • $i_1(0^+) = i(0^+) - i_2(0^+)$ (1)

les condensateurs étant initialement déchargés on a $\begin{cases} u_1(0^+) = u_1(0^-) = 0 \\ u_2(0^+) = u_2(0^-) = 0 \end{cases}$



Donc (5) ⇒ $i_2(0^+) = 0$

(1) ⇒ $i_1(0^+) = i(0^+)$

(4) ⇒ $E - R\, i(0^+) = 0$ $\Big\}$ $\boxed{i_1(0^+) = i(0^+) = \frac{E}{R}}$

• $\frac{di_1}{dt}(0^+)$? (*) ⇒ $3 i_1(0^+) + CR \frac{di_1}{dt}(0^+) + \frac{\cancel{q_1(0^+)}}{RC} = \frac{E}{R}$

↳ $\frac{di_1}{dt}(0^+) = \frac{E}{CR^2} - 3\frac{E}{CR^2} \longrightarrow \boxed{\frac{di_1}{dt}(0^+) = -\frac{2E}{CR^2}}$

3°) Résolution de l'équation différentielle :

éq caractéristique associée : $r^2 + \frac{3}{RC} r + \frac{1}{R^2C^2} = 0$

$\Delta = \frac{9}{R^2C^2} - \frac{4}{R^2C^2} = \frac{5}{R^2C^2} > 0$ → 2 racines réelles négatives $\begin{cases} r_1 = \frac{1}{2RC}(-3-\sqrt{5}) < 0 \\ r_2 = \frac{1}{2RC}(-3+\sqrt{5}) < 0 \end{cases}$

$$i_1 = A \exp\left(\frac{-3-\sqrt{5}}{2RC} t\right) + B \exp\left(\frac{-3+\sqrt{5}}{2RC} t\right) = \exp\frac{-3t}{2RC}\left(A \exp\frac{-\sqrt{5}}{2RC} t + B \exp\frac{+\sqrt{5}}{2RC} t\right)$$

① $i_1(0^+) = \frac{E}{R} = A + B$

$$\frac{di_1}{dt} = \frac{-3}{2RC}\exp\frac{-3t}{2RC}\left(A\exp\frac{-\sqrt{5}t}{2RC} + B\exp\frac{\sqrt{5}t}{2RC}\right) + \frac{\sqrt{5}}{2RC}\exp\frac{-3t}{2RC}\left(-A\exp\frac{-\sqrt{5}}{2RC}t + B\exp\frac{\sqrt{5}}{2RC}t\right)$$

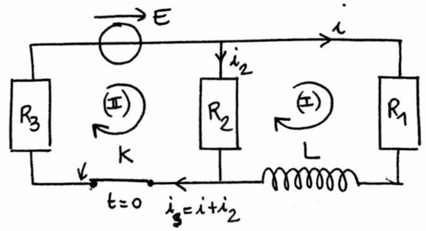
② $\frac{di_1}{dt}(0^+) = \frac{-2E}{CR^2} = \frac{-3}{2RC}(A+B) + \frac{\sqrt{5}}{RC}(-A+B)$ ←

$\begin{cases} ① \; A + B = \frac{E}{R} \\ ② \; \frac{-3}{2}\frac{E}{R} + \frac{\sqrt{5}}{2}(-A+B) = -\frac{2E}{R} \end{cases} \Leftrightarrow \begin{cases} ① \; A + B = \frac{E}{R} \\ ② \; A - B = \frac{E}{\sqrt{5}R} \end{cases}$ $\quad$ ①+② ⇒ $A = \frac{E}{2R}\left(1 + \frac{1}{\sqrt{5}}\right)$, $\quad B = \frac{E}{2R}\left(1 - \frac{1}{\sqrt{5}}\right)$

$$i_1(t) = \exp\frac{-3t}{2RC}\frac{E}{2R}\left[\exp\frac{-\sqrt{5}}{2RC}t + \exp\frac{+\sqrt{5}}{2RC}t + \frac{1}{\sqrt{5}}\left(\exp\frac{-\sqrt{5}}{2RC}t - \exp\frac{\sqrt{5}}{2RC}t\right)\right] \rightarrow \boxed{i_1(t) = \frac{E}{R}\exp\frac{-3t}{2RC}\left[\mathrm{ch}\frac{\sqrt{5}}{2RC}t - \frac{1}{\sqrt{5}}\mathrm{sh}\frac{\sqrt{5}}{2RC}t\right]}$$

EXE3.7 3 Résistances & 1 Bobine

1) Conditions initiales : $i(0^-) = 0$



loi des mailles dans (I) :

$-R_1 i - L\frac{di}{dt} + R_2 i_2 = 0$ (1)

loi des mailles dans (II) :

$-R_2 i_2 - R_3(i + i_2) + E = 0$ (2)

(1) → $i_2 = \frac{1}{R_2}\left(R_1 i + L\frac{di}{dt}\right)$ (3) $\xrightarrow{(2)}$ $\frac{R_2+R_3}{R_2}\left(R_1 i + L\frac{di}{dt}\right) + R_3 i = E$

soit $(R_2+R_3)L\frac{di}{dt} + \left(R_2R_3 + R_1(R_2+R_3)\right) i = E R_2$

soit $\frac{di}{dt} + \frac{R_1R_2 + R_1R_3 + R_2R_3}{L(R_2+R_3)} i = \frac{ER_2}{(R_2+R_3)L} \Leftrightarrow \frac{di}{dt} + \frac{1}{\tau} i = \frac{ER_2}{(R_2+R_3)L}$ (E)

avec $\tau = \frac{L(R_2+R_3)}{R_1R_2 + R_2R_3 + R_3R_1}$

d'où $i = i_G + i_P$

avec $i_G \equiv$ solution générale de (E) sans second membre $= A\exp\frac{-t}{\tau}$

$i_P \equiv$ solution particulière de (E) $= \frac{E\tau R_2}{(R_2+R_3)L} = I$

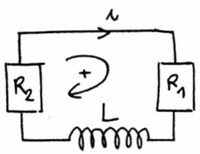
d'où $\forall\, t \geq 0^+$ $\quad i = A\exp\frac{-t}{\tau} + I_0$.

Comme i est une fonction continue du temps (puisque c'est l'intensité traversant une bobine)

on a $i(0^+) = \begin{cases} i(0^-) = 0 \\ A + I_0 \end{cases}$ d'où $\boxed{i = I_0\left(1 - \exp\frac{-t}{\tau}\right)}$ avec $\boxed{I = \frac{ER_2}{R_1R_2 + R_2R_3 + R_3R_1}}$ ←

lorsque $t \to \infty$ $\quad i(t) \to I$, intensité du courant permanent dans L.

2) • Lorsqu'on ouvre l'interrupteur, le circuit devient :



L'équation différentielle régissant $i(t)$ est alors :

$\frac{di}{dt} + \frac{(R_1+R_2)}{L} i = 0$ soit $\frac{di}{dt} + \frac{i}{\tau'} = 0$

d'où $i(t) = B\exp\frac{-t}{\tau'}$. À $t = 0^+$ $\; i(0^+) = B = i(0^-) = I$ (après réinitialisation de t)

d'où $\boxed{i(t) = I\exp\frac{-t}{\tau'}}$ avec $\boxed{\tau' = \frac{L}{R_1+R_2}}$ $\quad I \xrightarrow{i(t)} 0$ lorsque $t \to \infty$

• loi des mailles : $L\frac{di}{dt} + (R_1+R_2) i = 0 \rightarrow \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + (R_1+R_2)i^2 = 0$ soit $\mathcal{P}_J = -\frac{d}{dt}\left(\frac{1}{2}Li^2\right)$

d'où "$\mathcal{E}_J$" $= \Delta\mathcal{E}_R = \int_0^\infty \mathcal{P}_J\, dt = \int_0^\infty -\frac{d}{dt}\left(\frac{1}{2}Li^2\right)dt = \int_I^0 -d\left(\frac{1}{2}Li^2\right) = 0 - \left(-\frac{1}{2}LI^2\right) \Rightarrow \boxed{\mathcal{E}_J = \frac{1}{2}LI^2}$