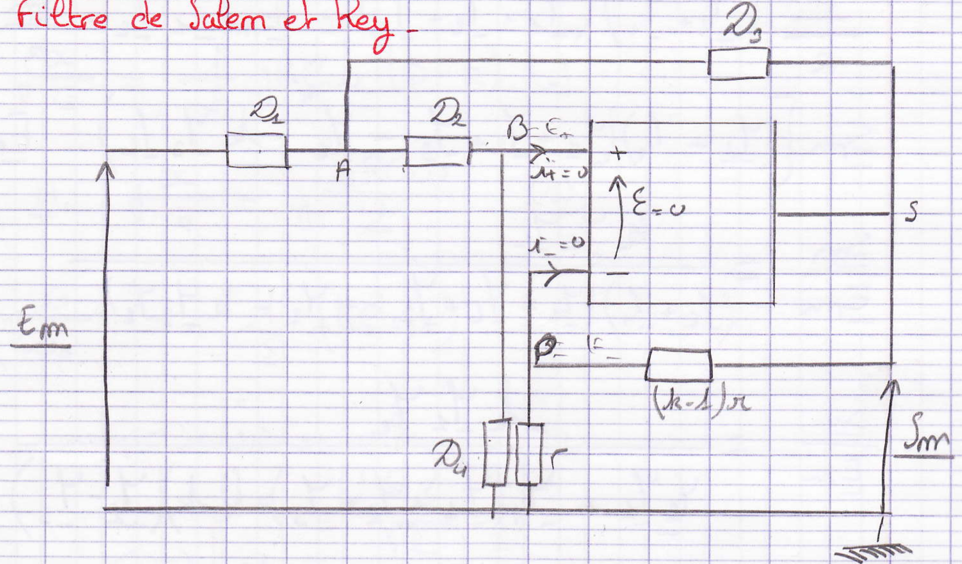
DM24 - Filtre actif et satellite -

I - Filtre de Salem et Key -



1) amplificateur operationnel idéal : $i_+ = i_- = 0$

régime linéaire $\begin{cases} \text{le gain de l'amplificateur tend vers l'infini} \\ \varepsilon = E_+ - E_- = 0 \end{cases} \iff \boxed{\underline{V}_P = \underline{V}_B}$ (1)

2) ~~théorème de Millman en A~~

$$\boxed{\underline{V}_A = \frac{\underline{E}_m \underline{Y}_1 + \underline{S}_m \underline{Y}_3 + \underline{V}_B \underline{Y}_2}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3}} \quad (2)$$

~~théorème de Millman en B~~

$$\underline{V}_B = \frac{\underline{V}_A \underline{Y}_2 + \cancel{\underline{V}_4 \underline{Y}_4} + \cancel{0}}{\underline{Y}_2 + \underline{Y}_4}$$

$$\boxed{\underline{V}_A = \frac{\underline{V}_B (\underline{Y}_2 + \underline{Y}_4)}{\underline{Y}_2}} \quad (3)$$

Pont diviseur de tension : $\underline{V}_P = \dfrac{r}{(k-1)r + r} \underline{S}_m$

$\underline{V}_P = \dfrac{1}{(k - \cancel{1}) \cancel{+ 1}} \underline{S}_m$ $\qquad \boxed{\underline{V}_P = \dfrac{\underline{S}_m}{k}}$ (4)

$$\xrightarrow[(3)\quad(4)]{(2)\quad(1)}$$

$$\underline{V}_A = \frac{1/k\,(\underline{Y}_2+\underline{Y}_4)\,S_m}{\underline{Y}_2} = \frac{E_m\,\underline{Y}_1 + 1/k\,S_m\,\underline{Y}_2 + V_s\,\underline{Y}_3}{\underline{Y}_1+\underline{Y}_2+\underline{Y}_3}$$

$$\frac{S_m}{k}(\underline{Y}_2+\underline{Y}_4)(\underline{Y}_1+\underline{Y}_2+\underline{Y}_3) = E_m\,\underline{Y}_1\underline{Y}_2 + \frac{S_m}{k}\underline{Y}_2^2 + S_m\,\underline{Y}_3\underline{Y}_2$$

$$S_m\left[(\underline{Y}_2+\underline{Y}_4)(\underline{Y}_1+\underline{Y}_2+\underline{Y}_3) - \underline{Y}_2^2 - k\,\underline{Y}_3\underline{Y}_2\right] = E_m\,k\,\underline{Y}_1\underline{Y}_2$$

$$\frac{S_m}{E_m} = \frac{k\,\underline{Y}_1\underline{Y}_2}{(\underline{Y}_2+\underline{Y}_4)(\underline{Y}_1+\underline{Y}_2+\underline{Y}_3) - \underline{Y}_2^2 - k\,\underline{Y}_3\underline{Y}_2}$$

$$\frac{S_m}{E_m} = \frac{k\,\underline{Y}_1\underline{Y}_2}{\underline{Y}_2\underline{Y}_1 + \underline{Y}_4(\underline{Y}_1+\underline{Y}_2+\underline{Y}_3) + (1-k)(\underline{Y}_2-\underline{Y}_3)}$$

$$\underline{H} = \frac{k\,\frac{1}{R}\,jC\omega}{j\frac{C\omega}{R} + \left(\frac{2}{R}+jC\omega\right)\left(\frac{1}{R}+jC\omega\right) + j\frac{C\omega}{R}(1-k)}$$

$$\underline{H} = \frac{k\,jRC\omega}{jRC\omega + (2+jRC\omega)(1+jRC\omega) + jRC\omega(1-k)}$$

$$\underline{H} = \frac{k\,jRC\omega}{jRC\omega + 2 + 2jRC\omega + jRC\omega + (jRC\omega)^2 + jRC\omega(1-k)}$$

$$\underline{H} = \frac{k\,jRC\omega}{2 + jRC\omega(5-k) + (jRC\omega)^2}$$

$$\boxed{\underline{H} = \frac{j\frac{kRC\omega}{2}}{1 + jRC\omega\frac{(5-k)}{2} + \frac{(jRC\omega)^2}{2}}}$$

Forme canonique associée :

$$\boxed{\underline{H} = \frac{H_0\,j\frac{x}{Q}}{1 + j\frac{x}{Q} + (jx)^2}}$$

Identification : $x = \frac{\omega}{\omega_0}$

$$\begin{cases} (jx)^2 = \frac{1}{2}(jRC\omega)^2 & (1) \\ j\frac{x}{Q} = \frac{1}{2} jRC\omega\,(5-k) & (2) \\ \underline{H}_0\ j\frac{x}{Q} = j\frac{kRC\omega}{2} & (3) \end{cases}$$

(1) $\left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{2}(RC\omega)^2$

$$\Leftrightarrow \omega_0^2 = \frac{2}{(RC)^2}$$

$$\Leftrightarrow \boxed{\omega_0 = \frac{\sqrt{2}}{RC}}$$

(2) $\frac{\omega}{\omega_0 Q} = \frac{1}{2}(RC\omega)(5-k)$

$$\boxed{Q = \frac{2}{\omega_0 RC(5-k)} = \frac{\sqrt{2}}{5-k}}$$

(3) $\underline{H}_0 \cancel{j}\frac{\cancel{\omega}}{\omega_0 Q} = \cancel{j}\frac{kRC\cancel{\omega}}{2} \Leftrightarrow \underline{H}_0 = \frac{Q\omega_0 kRC}{2}$

$$\Leftrightarrow \boxed{\underline{H}_0 = \frac{k}{5-k}}$$